# Cambridge International A Level

**MATHEMATICS** **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.



## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

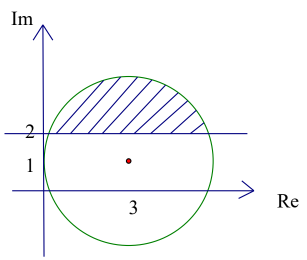
**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Use exponentials or law for the logarithm of a product, quotient or power | **M1*** | $e^{\ln(5+x)} = e^{5+\ln x}$ insufficient.<br>Need e.g. $\ln\left(\frac{x+5}{x}\right) = 5$ or $\ln(x+5) = \ln(e^5) + \ln x$<br>or $\ln(x+5) = \ln(e^5 x)$<br>or $x + 5 = e^{5+\ln x}$ or $x + 5 = e^5 e^{\ln x}$ and others. |
| | Correctly remove logarithms | **DM1** | |
| | Obtain a correct equation in $x$ | **A1** | e.g. $\frac{x+5}{x} = e^5$ (or 148.4…) or $x + 5 = xe^5$. |
| | Obtain 0.034 | **A1** | CAO Final answer must be 3d.p. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Divide to obtain quotient $2x^2 \pm 2x + k$ $(k \neq 0)$ | **M1** | Obtain result in answer column, together with a linear polynomial or a constant as remainder.<br>If correct:<br>$x^2+x+3 \,\overline{)\,2x^4 \qquad\qquad -27}$ with quotient $2x^2-2x-4$<br>$2x^4+2x^3+6x^2$<br>$-2x^3-6x^2$<br>$-2x^3-2x^2-6x$<br>$-4x^2+6x-27$<br>$-4x^2-4x-12$<br>$10x-15$ |
| | Obtain [quotient] $2x^2 - 2x - 4$ | **A1** | Allow unless quotient and remainder interchanged, then A0 A1. |
| | Obtain [remainder] $10x - 15$ | **A1** | Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$. |
| | **Alternative Method for Question 2** | | |
| | Expand $\left(x^2+x+3\right)\left(Ax^2+Bx+C\right)+\left(Dx+E\right)$ and reach $A = 2$, $B = \pm 2$, $C = k$ | **M1** | Solve all 3 equations for $A$, $B$ and $C$, allow sign errors in establishing equations and in solving.<br>If correct, $A = 2$, $A + B = 0$, $3A + B + C = 0$, $3B + C + D = 0$, $3C + E = -27$.<br>Obtain result in answer column, together with a linear polynomial or a constant as remainder. |
| | Obtain [quotient] $2x^2 - 2x - 4$ | **A1** | Allow unless quotient and remainder interchanged, then A0 A1. |
| | Obtain [remainder] $10x - 15$ | **A1** | Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Show a circle with centre $3+\mathrm{i}$ | **B1** | Must be some evidence of scale on both axes or centre stated as 3 + i or (3, 1). |
| | Show a circle with radius 3 and centre not at the origin | **B1** | Must be some evidence that radius = 3 or stated $r = 3$ |
| | Show the line $y=2$ | **B1** | Line $y = 2$ can be represented by 2 or correct dashes. |
| | Shade the correct region | **B1** | Line and circle must be correct. |
| |  | **4** | Scales may be replaced by dashes on axes for all marks.<br>Correct figure, with no scale on either axis then allow 1/3 and the B1 for correct shaded region Max 2/4.<br>If B0 above for line but relatively correct position then B1 for correct shaded region Max 3/4.<br>Re and Im axes interchanged but clearly labelled, allow **SCB1** for centre and radius of circle correct and **SCB1** for line and shading correct Max 2/4. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | State $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 1 - 2\sin\theta$ | **B1** | Ignore left side throughout d$x$/d$t$, d$y$/d$t$, d$x$, d$y$ but must see $\frac{\mathrm{d}y}{\mathrm{d}x}$ for final A1. |
| | Use correct quotient rule, or product rule if rewrite $x$ as $\cos\theta(2-\sin\theta)^{-1}$ | **M1** | Incorrect formula seen M0 A0 otherwise BOD. |
| | Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{-(2-\sin\theta)\sin\theta + \cos^2\theta}{(2-\sin\theta)^2}$ o.e. | **A1** | $-\sin\theta(2-\sin\theta)^{-1} - \cos\theta(2-\sin\theta)^{-2}(-\cos\theta)$ or equivalent. |
| | Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta}$ | **M1** | $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = (1-2\sin\theta) \div \frac{1-2\sin\theta}{(2-\sin\theta)^2}\right)$. Allow M1 even if errors in both derivatives. |
| | Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = (2-\sin\theta)^2$. | **A1** | AG – must see working in above cell to gain final A1. Allow $\cos^2\theta + \sin^2\theta = 1$ to be implied. $x$ instead of $\theta$ or missing $\theta$ more than twice on right side then A0 final mark. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Use correct product rule | **M1** | $\frac{\mathrm{d}}{\mathrm{d}x}(x^2)\cos(3x)+x^2\frac{\mathrm{d}}{\mathrm{d}x}(\cos 3x)$. |
| | Obtain correct derivative in any form | **A1** | e.g. $2x\cos 3x - 3x^2\sin 3x$ . |
| | Equate derivative to zero and obtain $a=\frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$. | **A1** | AG Condone $a=\frac{1}{3}\tan^{-1}\frac{2}{3a}$.<br>Must at least reach expression $2x = 3x^2\tan(3x)$ or better <u>before</u> final answer to gain A1.<br>Final answer must be in terms of $a$. Can work with $x$ and switch to $a$ at very end.<br>Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0. |
| | | **3** | |
| 5(b) | Use the iterative process $a_{n+1}=\frac{1}{3}\tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations | **M1** | Degrees 0/3. |
| | Obtain final answer 0.36 | **A1** | Must be 2d.p. |
| | Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval $(0.355, 0.365)$ | **A1** | Allow small errors in 4th d.p.<br>Allow errors at start if self corrects later. |
| | 0.5 0.4 0.3 0.2 0.1 π/6 π/12<br>0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989<br>0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439<br>0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649<br>0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567<br>0.3578 0.3604 0.3614 0.3576<br>0.3580 | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Expand $\cos(x-60°)$ correctly and evaluate $3\cos x + 2\cos(x - 60°)$ to obtain $4\cos x+\sqrt{3}\sin x$ or unsimplified coefficients | **B1** | Need to see $A\cos x + B\sin x$ with $A$ and $B$ correct<br>$A$ may be 4 or 3 + 2 cos 60 and $B$ may be $\sqrt{3}$ or 2 sin 60. |
| | State $R=\sqrt{19}$ [$R\cos\alpha = 4$ $R\sin\alpha = \sqrt{3}$ ] | **B1 FT** | Follow through *their* 4 and $\sqrt{3}$ .<br>If coefficients are 3 and 2 then B0.<br>$R = \sqrt{19}$ from $R$ = 4.36 B0<br>but 4.36 seen after $\sqrt{19}$ ISW. |
| | Use correct trig formulae for their expansion to find $\alpha$<br>e.g. $\alpha = \tan^{-1}\dfrac{\sqrt{3}}{4}$ or $\cos^{-1}\dfrac{4}{\sqrt{19}}$ or $\sin^{-1}\dfrac{\sqrt{3}}{\sqrt{19}}$ | **M1** | If $\sin\alpha = \sqrt{3}$ $\cos\alpha = 4$ seen then M0 A0.<br>If $\underline{\tan\alpha} = 23.41°$ M0 A0 but can recover if $\alpha = 23.41°$ seen later.<br>$\alpha = \tan^{-1}\dfrac{2}{3}$ M1 $(\alpha = 33.69°)$ but $\alpha = \tan^{-1}\dfrac{3}{2}$ M0 |
| | Obtain $\alpha = 23.41°$ | **A1** | Allow if $x$ instead of $\alpha$. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $\cos^{-1}\left(\dfrac{2.5}{R}\right)$ | **B1 FT** | SOI [55.0°].<br>Follow through *their* $\sqrt{19}$ . |
| | Use a correct method to find a value of $2\theta$ (not $x$) in the interval.<br>Allow sign error in moving $\alpha$ to right side | **M1** | $2\theta = \cos^{-1}\left(\dfrac{2.5}{R}\right) + 23.41°$<br>or $2\theta = 360° - \cos^{-1}\left(\dfrac{2.5}{R}\right) + 23.41°$<br>with $R$ substituted. |
| | Obtain one correct answer e.g. $39.2°$ | **A1** | If working for **M1** not seen then **M1** implied by $39.2°$ or $164.2°$<br>Must be at least 1d.p. |
| | Obtain second correct answer e.g. $164.2°$ and no others in the interval | **A1** | Must be at least 1d.p.<br>Ignore answers outside the given interval. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$ | **B1** | SOI |
| | Use double angle formula and substitute for $x$ and d$x$ throughout the integral | **M1** | All $x$'s must be removed, can be coefficient errors provided 2 seen in working. |
| | Obtain $\pm\int 2u\mathrm{e}^{2u}\,\mathrm{d}u$ | **A1** | Limits may be omitted, or left as 0 and π, during the change of variable stage. |
| | Justify new limits and obtain $\int_{-1}^{1} 2u\mathrm{e}^{2u}\,\mathrm{d}u$ from correct working | **A1** | AG Must see $x = 0$, $u = 1$ and $x = \pi$, $u = -1$.<br>Inequalities alone e.g. $0 \leqslant x \leqslant \pi$ and $1 \leqslant u \leqslant -1$ or $-1 \leqslant u \leqslant 1$ for limits are insufficient A0<br>If sign in expression and order of limits incorrect then A0.<br>If negative sign is present in the integrand then this can be removed and limits introduced in correct order in a single step. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | Commence integration and reach $aue^{2u} + b\int e^{2u}\,du$, where $ab \ne 0$, $b < 0$ | M1* | Condone d$x$. |
| | Complete integration and obtain $ue^{2u} - \frac{1}{2}e^{2u}$ | A1 | OE Allow $(2u\frac{1}{2}e^{2u}) - \frac{1}{2}e^{2u}$. |
| | Use correct limits correctly in $c\,ue^{2u} + d\,e^{2u}$ having integrated twice<br><br>or in $c\cos x\,e^{2\cos x} + d\,e^{2\cos x}$ | DM1 | 1 and −1 for $u$, 0 and $\pi$ for $x$<br>e.g. $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. Not decimals.<br>Allow one sign error at most in going from<br>$c\,ue^{2u} + d\,e^{2u}$ or $c\cos x\,e^{2\cos x} + d\,e^{2\cos x}$ to<br>$ce^2 + de^2 - (-ce^{-2} + de^{-2})$.<br>$[e^2 - ½\,e^2 - (-e^{-2} - ½\,e^{-2})]$<br>Complete reversal of sign by converting back to cos $x$ and not making $x = 0$ upper limit is DM0 A0. |
| | Obtain $\frac{1}{2}e^2 + \frac{3}{2}e^{-2}$ | A1 | ISW<br>Or equivalent 2-term expression e.g. $\frac{e^4+3}{2e^2}$ or<br>$\frac{1}{2}\left(e^2 + \frac{3}{e^2}\right)$. |
| | | 4 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8 | Separate the variables correctly | B1 | $\dfrac{y+4}{y^2+4}\,\mathrm{d}y = \dfrac{1}{x}\,\mathrm{d}x$. |
| | Obtain $\ln x$ | B1 | |
| | Split the fraction and integrate to obtain $p\ln\left(y^2+4\right)$ or $q\tan^{-1}\frac{y}{2}$ correctly | *M1 | Only following subdivision into $\dfrac{y}{y^2+4}+\dfrac{4}{y^2+4}$. If no subdivision seen then both terms $p\ln\left(y^2+4\right)$ and $q\tan^{-1}\dfrac{y}{2}$ must be present. |
| | Obtain $\frac{1}{2}\ln\left(y^2+4\right)$ | A1 | |
| | Obtain $2\tan^{-1}\frac{y}{2}$ | A1 | |
| | Use $\left(4, 2\sqrt{3}\right)$ in an expression containing at least 2 of $a\ln x, b\ln\left(y^2+4\right)$ and $c\tan^{-1}\frac{y}{2}$ to obtain constant of integration | DM1 | Allow one sign or arithmetic error e.g. $\dfrac{2\pi}{3}$. May use $\left(4, 2\sqrt{3}\right)$ and $(x, 2)$ as limits to find $x$ for the final 3 marks. |
| | Correct solution (any form) e.g. $\dfrac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\dfrac{y}{2}=\ln x+\dfrac{2\pi}{3}$ or $\dfrac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\dfrac{y}{2}=\ln x+2\tan^{-1}\sqrt{3}+\dfrac{1}{2}\ln 16-\ln 4$ | A1 | However solution not asked for so allow $\dfrac{1}{2}\ln 8+2\tan^{-1}1=\ln x+2\tan^{-1}\sqrt{3}+\dfrac{1}{2}\ln 16-\ln 4$. |
| | Obtain $\sqrt{8}\mathrm{e}^{-\frac{1}{6}\pi}$ or 1.68 or more accurate or $2\sqrt{2}\mathrm{e}^{-\frac{1}{6}\pi}$ or $\dfrac{\sqrt{8}}{\mathrm{e}^{\frac{1}{6}\pi}}$ or $\mathrm{e}^{0.516}$ | A1 | ISW Must remove ln so $x = \mathrm{e}^{(\ln 2\sqrt{2} - \pi/6)}$ A0. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8 | **Alternative method for first *M1 A1 A1** | | |
| | $p\left((y+4)\tan^{-1}\dfrac{y}{2}-\int\tan^{-1}\dfrac{y}{2}\,\mathrm{d}y\right)$ | *M1 | Allow sign error. |
| | $(y+4)\dfrac{1}{2}\tan^{-1}\dfrac{y}{2}-\dfrac{y}{2}\tan^{-1}\dfrac{y}{2}+\int\dfrac{y}{y^2+4}\,\mathrm{d}y$ | A1 | |
| | Obtain $2\tan^{-1}\frac{y}{2}+\frac{1}{2}\ln\left(y^2+4\right)$ | A1 | |
| | | 8 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | Perform scalar product of direction vectors and set result equal to zero | **M1** | $2c+6+4=0$ . |
| | Use $P$ to find the value of $\lambda$ | **M1** | $3-2\lambda=7 \Rightarrow \lambda=-2$ [$a+\lambda$c = 4, $b+4\lambda=-2$].<br>Equation for line $l$ may contain $-\lambda$ instead of $+\lambda$ leading to $\lambda=2$ all marks available. |
| | Obtain $c=-5$ or $b=6$ | **A1** | |
| | $a=-6$, $b=6$ and $c=-5$ all correct | **A1** | |
| | | **4** | **SC1**: Use $P$ to find the value of $\lambda$ M1<br>Substitute $\lambda=-2$ into point $P$, so $a-2c=4$, and put $\mu=-1$ and $\lambda=-1$ into $l$ so $a-c=-1$, then solve to obtain $a=-6$ , $b=6$ and $c=-5$.<br>All 3 values correct A1.<br>Max 2/4. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(b) | Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point $Q$ on $m$<br>$= \pm((1+2\mu, 2-3\mu, 3+\mu)-(a+\lambda c, 3-2\lambda, b+4\lambda))$ | **B1** | $\left[\overrightarrow{PQ} \text{ or } \overrightarrow{QP} = \pm\begin{pmatrix}-3+2\mu\\-5-3\mu\\5+\mu\end{pmatrix}\right]$<br>Could be *their* $a$, $b$, $c$ and $\lambda$ values provided M1 M1 gained in **(a)**.<br>Allow expression in answer column. |
| | Equate the scalar product of $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) and a direction vector for $m$ to zero and obtain an equation in $\mu$ | **M1*** | $(2(-3+2\mu)-3(-5-3\mu)+(5+\mu))=0$.<br>Allow $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$ sign problem. |
| | Solve and obtain $\mu=-1$ | **A1** | $PQ^2 = (-3+2\mu)^2+(-5-3\mu)^2+(5+\mu)^2$.<br>$[= 14(\mu+1)^2+45]$.<br>Min when $\mu = -1$ or by differentiation. |
| | Obtain $\overrightarrow{OQ} = -\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ or $\overrightarrow{PQ} = -5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$<br>Must be labelled correctly | **A1** | The working may be in **(a)** provided at least this result is used in **(b)**. |
| | Carry out a method to find the position vector of $R$<br><br>**Alternative method for DM1**<br>$\overrightarrow{OR} = (4, 7, -2) + t(-5, -2, 4)$ $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$<br>Solve $\lvert\boldsymbol{QR}\rvert^2 = \frac{9}{4}\lvert\boldsymbol{PQ}\rvert^2$ or $\lvert\boldsymbol{QR}\rvert = \frac{3}{2}\lvert\boldsymbol{PQ}\rvert$ $t = 2.5$ | **DM1** | e.g. Use $\overrightarrow{OR} = \overrightarrow{OP} + \frac{5}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \overrightarrow{OQ} + \frac{3}{2}\overrightarrow{PQ}$ or<br>$\overrightarrow{OR} = \frac{5}{2}\overrightarrow{OQ} - \frac{3}{2}\overrightarrow{OP}$ or $2\overrightarrow{QR} = 2(\overrightarrow{OR} - \overrightarrow{OQ}) = 3\overrightarrow{PQ}$<br>where $\overrightarrow{OR} = (x, y, z)$.<br>$\overrightarrow{PQ}$ used in all these approaches, may be incorrect, must be in the correct direction, i.e. not using $\overrightarrow{QP}$ for $\overrightarrow{PQ}$. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(b) | Obtain $-\frac{17}{2}\boldsymbol{i}+2\boldsymbol{j}+8\boldsymbol{k}$ from correct working | **A1** | Accept coordinates.<br>Don't accept $-\frac{17}{2}\mathbf{i}+\frac{4}{2}\mathbf{j}+\frac{16}{2}\mathbf{k}$. |
| | | **6** | **SC2** Equate lines, attempt to find $\mu=-1$ or $\lambda=-1$ M1*<br>$\overrightarrow{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ A1.<br>Attempt to find $\overrightarrow{OQ}$ using other parameter value DM1.<br>$\overrightarrow{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ therefore intersect A1.<br>Then use main scheme for the final DM1 A1. |
| | | | First **DM1 A1** are available if they show the 3 coordinates are consistent for the 2 parameter values instead of attempting to find $\overrightarrow{OQ}$ using the other parameter value and then showing intersection |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | State or imply the form $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{C}{(3-x)^2}$ | **B1** | Alternative form: $\frac{A}{1+2x}+\frac{Dx+E}{(3-x)^2}$. |
| | Use a correct method to find a constant | **M1** | Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values.<br>Max 2/5<br>Allow M1 even if multiply up by $(1 + 2x)(3 - x)^3$. |
| | Obtain one of $A = 2$, $B = 2$ and $C = -3$ | **A1** | Alternative form: obtain one of $A = 2$, $D = -2$ and $E = 3$. |
| | Obtain a second value | **A1** | |
| | Obtain the third value | **A1** | Do not need to substitute values back into original form. |
| | | **5** | If $\frac{A}{1+2x}+\frac{B}{3-x}+\frac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for *A*, A1 for *B* and A1 for *C* and *D*. If $C = 0$ then recovers B1 from above. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | Use a correct method to obtain the first two terms of one of the unsimplified expansions $(1+2x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}, \left(1-\frac{1}{3}x\right)^{-2}$ $(3-x)^{-1}$, $(3-x)^{-2}$ | M1 | $(1+2x)^{-1} = 1 + (-1)(2x) + \ldots$<br>$\left(1-\frac{1}{3}x\right)^{-1} = 1 + (-1)(-x/3) + \ldots$<br>$\left(1-\frac{1}{3}x\right)^{-2} = 1 + (-2)(-x/3) + \ldots$<br>$(3-x)^{-1} = 3^{-1} + (-1)3^{-2}(-x) \ldots$<br>$(3-x)^{-2} = 3^{-2} + (-2)3^{-3}(-x) + \ldots$ |
| | Obtain the correct unsimplified expansions up to the term in $x^2$ for each partial fraction<br><br>If correct, should be working with<br>$\frac{2}{1+2x}+\frac{2}{3-x}-\frac{3}{(3-x)^2}$ or $\frac{2}{1+2x}+\frac{-2x+3}{(3-x)^2}$ | A1 FT<br>A1 FT<br>A1 FT | Follow through on *their* $A$, $B$, $C$<br>$A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \ldots)$<br>$\frac{B}{3}$ $(1 + (-1)(-x/3) + ((-1)(-2)/2)(-x/3)^2 + \ldots)$<br>$\frac{C}{3^2}$ $(1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + \ldots)$.<br>Must be *their* coefficients from **(a)** but may be unsimplified expansions for FT marks.<br>If correct, expect to see<br>$2(1 - 2x + (2x)^2)$ or $2-4x+8x^2$<br>$\frac{2}{3}$ $\left(1 + \frac{x}{3} + \left(\frac{x}{3}\right)^2\right)$ or $\frac{2}{3}+\frac{2}{9}x+\frac{2}{27}x^2$<br>$-\frac{1}{3}$ $\left(1 + \frac{2x}{3} + (3)\left(\frac{x}{3}\right)^2\right)$ or $-\frac{1}{3}-\frac{2}{9}x-\frac{x^2}{9}$. |
| | Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$ | A1 | Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | **Alternative Method for Question 10(b)** | | |
| | For the form $\frac{A}{1+2x}+\frac{Dx+E}{(3-x)^2}$ | **M1*** | For the first two terms of an expanded partial fraction, following their $A$, $D$, $E$. |
| | Obtain the correct unsimplified expansions up to the term in $x^2$ for each partial fraction | **A1FT** | $A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \ldots) +$ |
| | | **A1FT** | $(Dx + E)\frac{1}{3^2}\,(1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + )$<br>$2(1 - 2x + (2x)^2 + \ldots)$<br>$+\ \frac{-2x+3}{3^2}\ (1 + \frac{2x}{3} + (3)\left(\frac{x}{3}\right)^2 + \ldots).$ |
| | Multiply out fully | **DM1** | Provided $DE \neq 0$.<br>Ignore cubic terms and above.<br>Allow error in one term but all terms must be present.<br>If correct, expect to see<br>$2-4x+8x^2-\frac{2}{9}x-\frac{4}{27}x^2+\frac{1}{3}+\frac{2}{9}x+\frac{1}{9}x^2$ |
| | Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$ | **A1** | Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | **Alternative Method for Question 10(b): Maclaurin's Series** | | |
| | Correct derivatives for $A(1+2x)^{-1}$, $B(3-x)^{-1}$ and $C(3-x)^{-2}$<br>$(-1)(2)A(1+2x)^{-2}$, $(-1)(-1)B(3-x)^{-2}$ and $(-2)(-1)C(3-x)^{-3}$ | **B1 FT** | |
| | One of following $(-2)(2)(-1)(2)A(1+2x)^{-3}$, $(-2)(-1)(-1)(-1)B(3-x)^{-3}$ and $(-3)(-1)(-2)(-1)C(3-x)^{-4}$ | **B1 FT** | |
| | All correct | **B1 FT** | |
| | Substitute in $f(0)+xf'(0)+\frac{x^2}{2}f''(0)$ | **M1** | |
| | Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$ | **A1** | Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | Multiply numerator and denominator by $(3-a\mathrm{i})$ | **M1** | Must perform complete multiplications but need not simplify $\mathrm{i}^2$. Can have errors but no term duplicated or missing.<br>$\dfrac{(5a-2\mathrm{i})(3-a\mathrm{i})}{9-a^2}=\dfrac{13a-i\left(5a^2+6\right)}{9-a^2}$ M0 M1 A0<br>No working so unsure if denominator multiplied by $3-a\mathrm{i}$ M1 M1 A0 |
| | Use $\mathrm{i}^2=-1$ at least once and separate real and imaginary parts | **M1** | |
| | Obtain $\dfrac{13a-\mathrm{i}\left(5a^2+6\right)}{9+a^2}$ or $\dfrac{13a-5a^2\mathrm{i}-6\mathrm{i}}{9+a^2}$ | **A1** | OE<br>If $15a-2a=13a$ seen later award this A1. |
| | Use $\arg z$ to form equation in $a$<br>$-\dfrac{5a^2+6}{13a}=\pm\tan\left(\pm\dfrac{\pi}{4}\right)$ or $-\dfrac{13a}{5a^2+6}=\pm\tan\left(\pm\dfrac{\pi}{4}\right)$<br>or $\tan^{-1}\left(-\dfrac{5a^2+6}{13a}\right)=\pm\dfrac{\pi}{4}$ or $\tan^{-1}\left(-\dfrac{13a}{5a^2+6}\right)=\pm\dfrac{\pi}{4}$ | **M1** | Allow expression given in answer column<br>or $5a^2+6=\pm13a$<br>or use $-(x\pm x\mathrm{i})=(13a-\mathrm{i}(5a^2+6))/(9+a^2)$<br>and eliminate $x$ so $5a^2+6=\pm13a$ M1. |
| | Obtain $a=2$ | **A1** | Need to reject $a=\dfrac{3}{5}$ or ignore it in future work.<br>May not see second root, but if present, must be $\dfrac{3}{5}$. |
| | Obtain $z=2-2\mathrm{i}$ only | **A1** | Allow $z=-2\mathrm{i}+2$. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | **Alternative Method 1 for the first four marks** | | |
| | arg $z$ = arg $(5a - 2\mathrm{i})$ – arg $(3 + a\mathrm{i})$ | **M1** | |
| | $= \tan^{-1}\left(\frac{-2}{5a}\right) - \tan^{-1}\left(\frac{a}{3}\right)$ <br> $= \tan^{-1}\left(\frac{-2}{5a} - \frac{a}{3}\right) / \left(1 + \left(\frac{-2}{5a}\right)\left(\frac{a}{3}\right)\right)$ | **M1** | Allow one sign error in second M1. |
| | $= \tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ or $\tan^{-1}\left(-\frac{13a}{5a^2+6}\right)$ | **A1** | |
| | $\pm\frac{\pi}{4} = \tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ or $\tan^{-1}\left(-\frac{13a}{5a^2+6}\right)$ | **M1** | Equate *their* $\tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ to $\pm\frac{\pi}{4}$. <br> Then as original scheme for final 2 marks. |
| | **Alternative Method 2 for the first four marks** | | |
| | $(x + \mathrm{i}y)(3 + a\mathrm{i}) = 5a - 2\mathrm{i}$ <br> $3x - ay = 5a$ and $ax + 3y = -2$ | **M1 A1** | |
| | $x = \pm y$ Find $x$ or $y$ in terms of $a$, e.g. $x = \frac{2}{3-a}$ or $x = \frac{5a}{3+a}$ | **M1** | |
| | Substitute in other equation, for example $3\left(\frac{2}{3-a}\right) + a\left(\frac{2}{3-a}\right) = 5a$ | **M1** | Then as original scheme for final 2 marks. |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(b) | State $\arg\left(z^3\right) = -\frac{3}{4}\pi$ or evaluate from $z = b - b\text{i}$ or from $-2b^3(1 + \text{i})$ | **B1** | If 2 different values given award B0.<br>Do not ISW. |
| | Complete method to obtain $r$ from *their z* | **M1** | $\left\|z^3\right\| = \left(\sqrt{x^2 + y^2}\right)^3$.<br>If $z$ correct, may see $\left\|z^3\right\| = \left(\sqrt{2^2 + (-2)^2}\right)^3$<br>or $\left\|z^3\right\| = \sqrt{(-16)^2 + (-16)^2}$. |
| | $r = 16\sqrt{2}$ | **A1** | CAO<br><br>A1 if $z = 2 - 2\text{i}$ obtained correctly.<br>or $z =$ used with $a = 2$ found correctly, otherwise A0XP.<br><br>May see arg and $r$ given in a final answer i.e. $16\sqrt{2}\text{e}^{-\frac{3}{4}\pi\text{i}}$.<br>Allow this form for arg and $r$ to collect full marks, even if i missing.<br>Ignore answers outside the given interval.<br>If 2 different values given award A0. |
| | | **3** | |